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RESEARCH

The Only Other Spending Rule Article You Will Ever Need

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This work describes an actionable framework for constructing and drawing income from a portfolio of retirement assets. A sufficient portfolio consists solely of a ladder of inflation-indexed bonds, such as U.S. Treasury Inflation-Protected Securities (TIPS), and a stock market index fund. We consider longevity risk and time-varying spending patterns and show how to amortize decumulation of the stock asset to provide variable income without risking premature portfolio depletion. We explain theoretically and demonstrate empirically how this strategy is less risky and more effective at maximizing lifetime retirement income than are methods commonly used by financial advisors.

Keywords: decumulation; financial planning; retirement income; spending rules; Treasury Inflation-Protected Securities

Disclosure: The author may develop services based on this research.

PL Credits: 2.0

Introduction

"How much income can I count on from my retirement savings?" This is a paramount question for any American with an Individual Retirement Account (IRA) or 401(k), and for those in other nations with a similar defined contribution account—especially for the growing majority without a defined benefit pension. It is a notoriously tricky question to answer due to the uncertainty of both longevity and market returns. Inflation-indexed life annuities would simplify the problem, but few if any such products exist in today's marketplace. Many articles have been written on the topic of spending down retirement assets, primarily from the perspective of avoiding premature portfolio depletion. But few have adequately explained how to satisfactorily avoid *both* the rock of outliving one's assets *and* the hard place of chronic underspending.

This work makes novel contributions to the body of knowledge on retirement asset decumulation. Our main contribution is a practical and actionable framework for both construction of the retirement portfolio and a strategy for spending down the portfolio during the retiree's lifetime. We propose and justify a portfolio consisting solely of an equity index fund and a ladder of inflation-indexed bonds, held to maturity, along with rules for determining the allocation between the bond ladder and equities. This work focuses on the experience of American retirees, examining scenarios where the indexed bonds are U.S. Treasury Inflation-Protected Securities (TIPS) and the risky assets are U.S. securities. The core concepts, however, generalize internationally. Many countries issue inflation-indexed sovereign bonds similar to TIPS,

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including the United Kingdom (Index-linked Gilts), Canada (Real Return Bonds), France (OATi), and Australia (Treasury Indexed Bonds).

We expand on ideas in existing literature to provide a spending formula that is flexible enough to adapt to a retiree's evolving circumstances, yet which can be implemented by a practitioner or sophisticated retiree using only a spreadsheet. We show both theoretically and empirically how this framework would offer a more efficient use of retirement assets to provide a more economically comfortable retirement for many retirees than do the methods most commonly used by practitioners today.

We build on Waring and Siegel (2015)'s[1] *annually recalculated virtual annuity* (ARVA) strategy for retirement income. An ARVA portfolio consists of a riskless asset—in practice a ladder of TIPS—and/or risky assets, such as stocks and conventional bonds. The TIPS ladder guarantees a pre-defined real dollar income stream. Withdrawals are made from the risky portion of the ARVA portfolio as if it is amortized like a fixed-term annuity. Unlike standard annuities, however, the time to maturity may be adjusted periodically and the periodic withdrawals vary with the market values of the risky assets.

ARVA has distinct advantages over retirement income strategies commonly recommended by practitioners. The most common strategies are based on fixed rate withdrawal rules, such as Bengen (1994)'s "4% Rule," sometimes adding "guardrails," such as from Guyton and Klinger (2006), to moderately decrease or increase income after market swings. Fixed rate strategies, even with guardrails, can potentially deplete a portfolio, with the retiree[2] outliving their assets. More likely, though, the strategy turns out to be overly conservative, leading the retiree to live below their means while accumulating a larger than intended legacy. In contrast with the prevailing fixed rate and guardrail rules, which are designed to have an acceptably low probability of premature portfolio depletion, an ARVA strategy essentially guarantees that the portfolio will not be depleted during the retiree's lifetime. At the same time, ARVA positions the retiree to spend more of their available wealth with more control over their legacy. While an ARVA enables more total withdrawals than today's methods that are typically described as "safe," that advantage comes with the trade-off that withdrawals vary from year to year with fluctuations in the risky asset values.

Waring and Siegel (2015)'s foundational work on ARVA leaves some open questions for further exploration, which we address here. First, there is the choice of asset allocation for the risky assets. Second is allocation between the risky portion of the portfolio and the TIPS ladder. Third is management of income volatility. Fourth is generalization of the model to produce time-varying vs. constant income goals. Fifth is management of longevity risk. We assess the range of choices using the historical record of U.S. stock market and bond returns between 1871 and 2023.

We show that the preferred risky asset portfolio consists solely of a low-fee U.S. broad stock market index fund, amortized with a discount rate of the long-term geometric mean real return on U.S. stocks (6.9%).[3] An ARVA retiree would likely hold both stocks and bonds, but the allocation to bonds would be entirely in the TIPS ladder, and not in conventional bonds or risky bond funds. The initial allocation between TIPS and stocks would depend on the retiree's need for an income floor to meet essential spending, their tolerance for the risk of income falling short of the floor some years, the size of their portfolio, their other income sources, and their desire for a legacy and/or assurance of income beyond the 30-year limit of current TIPS ladders. We compare our approach against the Guyton and Klinger (2006) "guardrail" strategy and show that our approach produces higher lifetime income with a lower risk of subpar withdrawals. Instead of providing fixed formulaic "guardrail" rules, we offer realistic and adaptable approaches for managing income volatility and mitigating longevity risk.

Literature Review

There is a considerable body of literature on withdrawal strategies for retirement spending. Pfau (2015) surveys a number of strategies and categorizes them as either Decision Rule Methods or Actuarial Methods. Decision Rule Methods tend to start with a fixed withdrawal rate and assume a fixed conservative time horizon; aim to reduce fluctuation in annual withdrawals, imposing "guardrails" to place ceilings and floors on distributions; and are designed around minimizing the probability of depleting the portfolio, accepting the trade-off of leaving a larger than intended legacy. Actuarial Methods, on the other hand, are designed to work more like a fixed-term annuity or amortizing loan, with a specified non-negative future value at the end of the term. Each withdrawal is determined by a fraction of current portfolio value given the remaining time horizon. Consequently, and unlike decision rule methods, the portfolio can never be depleted, although withdrawals may become very small. Unlike loans and fixed annuities whose terms are set at underwriting, the time horizons under an Actuarial Method may be adjusted dynamically; if the portfolio includes volatile assets, the withdrawals will also be volatile. Income volatility is accepted as a trade-off for the high likelihood for higher total income during retirement and the low likelihood of a larger than intended legacy. Guardrail rules are less likely to be employed.

The Decision Rule Methods that Pfau (2015) considers include Bengen (1994)'s constant inflation-adjusted spending (the "4% rule"); Bengen (2001)'s fixed-percentage and floor-and-ceiling withdrawals; Guyton and Klinger (2006)'s guardrail rules, which we discuss in detail in a later section; and Zolt (2013)'s target percent adjustments. The reviewed Actuarial Methods include Waring and Siegel (2015); Frank, Mitchell, and Blanchett (2011, 2012a, 2012b)'s age-based three-dimensional model; Blanchett, Kowara, and Chen (2012)'s mortality-updating constant probability of failure and RMD rule; Blanchett (2013)'s simple formula.

Sharpe (2017) proposes the "Lockbox" strategy. At the outset some quantity of TIPS and/or risky assets are placed in a "lockbox" mental account for each year of the retirement period. The assets in each year's lockbox are to provide income for that year and the lockbox contents are left untouched until the box is "opened" in its designated year. Each year's income target is computed by a utility function incorporating the retiree's risk aversion, taking into account longevity probabilities. The value placed in each lockbox at inception is the present value of the income target. While the framing of the lockbox strategy is somewhat different than that of ARVA, under shared assumptions they are equivalent. The lockbox TIPS comprise a ladder which may be the same as in an ARVA. Given the same initial portfolio of risky assets, time horizon, discount rate and income targets, it is a straightforward application of the amortization equation to show that the lockbox and ARVA withdrawals would be identical. ARVA relaxes the constraint that the targets and matching assets are fixed from the outset, but rather can be reformed at any time in the presence of new conditions.

Kaplan (2020) also starts with the ARVA model but sets the amortization rate for the risky assets to be a function of the certainty equivalent return of the portfolio and the retiree's subjective discount rate and elasticity of intertemporal substitution. The resulting withdrawal rate is further adjusted to decline with survival probability. The actual withdrawal is to be the sum of nondiscretionary spending plus discretionary spending. The latter is the withdrawal rate applied to the sum of financial wealth plus present value of human capital (including future Social Security and other non-portfolio income) less present value of future nondiscretionary spending.

Kobor and Muralidhar (2020) propose a dynamic asset allocation strategy with the acronym (GLIDeS). A GLIDeS portfolio consists of both equities and hypothetical risk-free bonds with the acronym (SeLFIES). SeLFIES would provide secure real income like a TIPS ladder, but the cashflows could have arbitrary starting and ending dates, whereas the time frame for currently available TIPS ladder cashflows is fixed from construction to at most 30 years out. The asset allocation between equities and SeLFIES is adjusted annually, where the risky portion is a decreasing function of the "actual funded status" (the ratio of current asset value to the present value of the sequence of future consumption targets). The authors' simulations show that in most scenarios their strategy produces higher incomes than both static 60/40 stock/bond allocations and target date fund glidepaths with declining equity allocations.

Blanchett (2023) divides the spending goal into inelastic "needs" and elastic "wants" and calculates a funded ratio for each. The denominator in each ratio is the present value of the respective sequence of planned expenditures adjusted for survival probability. The numerator for the needs funded ratio is the current portfolio value plus the present value of future non-portfolio incomes adjusted for survival probability. The numerator for the wants funded ratio is the needs numerator less present value of future needs liabilities, with a floor of 0. The annual withdrawal for each goal may increase (decrease) per a formula of how far the funded ratio is greater (less) than 1.

Appendix A contains a table which summarizes the features of the aforementioned articles on retirement decumulation strategies.

Data and Methodology

We assess security market returns and retirement income streams using historical simulations. Our data source is Shiller (2023)'s publicly available file of U.S. stock and bond market monthly returns from January 1871 to June 2023. We divide the historical time frame into overlapping consecutive 360-month periods starting each month from January 1871 through July 1993, and simulate withdrawal strategies over the resulting 1,470 periods. We use the word "run" to indicate a period's sequence of returns. All reported outcomes are inflation adjusted. Details on the data and simulation algorithm are given in Appendix B.

Taxes and management fees are not considered here. Transaction costs to implement an ARVA portfolio are negligible. TIPS are available at major online brokerages with minimal trading costs and no custodial fees. The risky component of the strategy can be implemented by index Exchange Traded Funds (ETFs) with expense ratios of 5 bps or less. Taxes and asset location are a worthy topic for future study.

When this study and other retirement research discuss "probabilities" of market-based outcomes, it should be explicitly noted that these should not be interpreted as forward-looking frequentist probabilities. The sequence of asset returns that a retiree experiences is not generated by a well-understood process under a steady-state probability distribution which can be algorithmically replicated. A "97% probability of success" for a retirement plan does not mean the same thing as the 97% probability of rolling a pair of dice and coming up with a 3 or higher. "Probabilities" in this context are equivalent to Bayesian priors informed by historical data or Monte Carlo simulations. They represent imprecise degrees of belief and should be communicated as such in non-technical English to the individuals whose retirement choices are informed by them.

We opted for historical over Monte Carlo simulations. The latter are grounded in history too, but solely by way of summary statistics. Nearly every example of a Monte Carlo simulation we have found in the relevant literature assumes that joint stock and bond returns are independent and identically distributed (i.i.d.) bivariate normal, an assumption that is inconsistent with the historical record.[4] As we discuss in a later section, the historical simulation can lead to meaningfully different conclusions than those reached under the assumption of i.i.d. bivariate normality. We assume that the long record of historical experience in a variety of market climates, with its unfiltered empirical distribution of returns sequences, is at least as reasonable as any proxy for the spectrum of possible future outcomes. The historical record also seems to be a more credible and reassuring foundation for a retiree's degree of belief.

Motivations for ARVA

The Perfect Withdrawal Rate and the Budget Constraint.

Blanchett, Kowara and Chen (2012) define the Sustainable Spending Rate (SSR) under perfect information as "the maximum constant income a retiree could have realized from the portfolio had he or she (or they) known the duration of the retirement period and annual returns as they were to be experienced in retirement, such that it depletes the portfolio to zero at time of death." It is expressed as a percentage of the initial portfolio value. Suarez, Suarez and Walz (2015) define a more general measure as the Perfect Withdrawal Amount (PWA) and derive the formula:

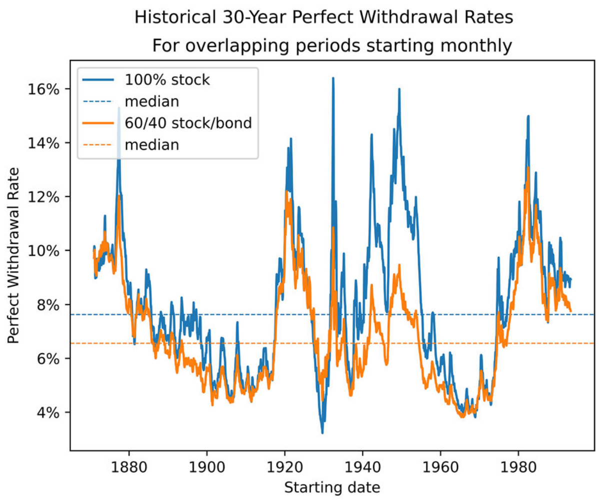
$$w = \frac{K_S \prod_{i=1}^{n} (1 + r_i) - K_E}{\sum_{i=1}^{n} \prod_{j=i}^{n} (1 + r_j)} \tag{1}$$

where K_S is the starting value of the portfolio, $K_E \geq 0$ is its ending value, and the r_i are the returns in the n periods between the start date and the end date. The formula depends on the *ordered* sequence of realized returns and assumes that the constant withdrawal amount is deducted from the portfolio every period. Suarez (2020) defines the Perfect Withdrawal Rate (PWR) as the PWA normalized by dividing the PWA by K_S. SSR is identical to PWR for the case $K_E = 0$.[5]

The ARVA method does not employ PWRs. But the PWR concept and the historical distribution of PWRs have important implications for any portfolio withdrawal strategy and help illuminate the distinctions between ARVA and Decision Rule Methods. Suarez (2020) calculates the PWR over historical time periods for portfolios with various allocations to stocks and bonds. Figure 1 illustrates the PWRs for the periods in our longer data set for 100% stock portfolios and for 60/40 stock/bond portfolios.

If a retiree attempted to make constant withdrawals during their retirement term at a rate above the term's PWR, they would deplete the portfolio prematurely

Figure 1. Timeline of Historical PWRs



(including the case of an incomplete final withdrawal). On the other hand, if the retiree were to make constant withdrawals below the PWR, they would leave a surplus in the portfolio. Consequently, if we define "probability of failure," as used in Decision Rule Methods, to be based on frequency of portfolio depletion in historical simulation for a given time horizon, then a constant withdrawal rate's percentile in the distribution of historical PWRs equals its failure probability for that time horizon.

Consider a 60/40 stock/bond portfolio. In the distribution of PWRs for the runs in our data, the 4% withdrawal rate was at the 2.0 percentile. The only runs where the 4% rule would have failed started during 1964–1968. In 98+% of runs the 4% Rule would have left the retiree with a surplus. The median run would have left the retiree with an inflation-adjusted legacy nearly 1.5x the initial portfolio value. While the 4% rule would have almost always ensured that a retiree never ran out of money in retirement, it would also have almost always denied the retiree as much income as their capacity would have allowed.

Pfau (2015) found that the guardrail rules of Guyton and Klinger (2006) and others would have permitted higher initial withdrawals than the 4% Rule and enabled the retiree to capture more of the upside while cushioning the shock of income reductions. But the PWR imposes some hard constraints. A retiree can spend more than the PWR some years, but not every year. There necessarily will be years when they spend less than the PWR. That might be because the withdrawal plan as designed happened to be below their PWR. In the worst case, they might embark on retirement spending at a rate above their PWR, which turns out to be historically low due to a large market drop early on. If they don't outlive their money altogether, they would have to reduce spending, and necessarily to a level below their already low PWR, at least in some years. Unless one improbably spends at exactly the PWR, it is impossible for there to be any withdrawal scheme or set of "guardrail" rules for a volatile portfolio that reliably delivers income that is free of (possibly large) reductions at unpredictable times, without also excessively preserving funds that one could otherwise spend in one's lifetime.

A related mathematical certainty is what Waring and Siegel (2018) call the "budget constraint": *At every point in time during the planning horizon the present value of planned future spending must be equal to the value of the assets*. Imposing a temporarily comforting but externally determined lower limit on today's withdrawal can mean taking more of today's present value than planned, and therefore a reduction in one's expected/planned future spending.

It is natural that retirees wish to (1) extract maximum value from their portfolio for spending during their lifetime, (2) be guaranteed that they don't outlive their savings, (3) ensure that their real income does not decline from one year to the next, and (4) control the amount of legacy they leave behind. Those goals cannot all be satisfied. A TIPS ladder is nearly riskless, but limited in time by the longest maturity of available bonds (currently 30 years), so it does not provide longevity protection.[6] Other financial assets are volatile. A near constant income stream with a near guarantee of lifetime sustainability would also come with a near guarantee of a lifetime of living below one's means. The more one wishes to withdraw from a volatile portfolio, the more volatile the income one must be prepared to accept.

A suitably constructed and managed ARVA portfolio can be adapted to the individual retiree's possibly time-varying preferences for the trade-offs between lower but stable income vs. higher expected but variable income. It can also adapt to changes in the retiree's consumption needs and beliefs about longevity.

Construction and Management of an ARVA Portfolio

Most retirement portfolios at present are composed primarily of equity funds and conventional bond funds. Our ARVA portfolio is also composed of equities and bonds, only different bonds than those most commonly used today. Waring and Siegel (2015) and Sharpe (2017) both use a TIPS ladder for the risk-free portion of the portfolio. The former does not specify the composition of the risky portion. The latter uses the entire market portfolio of global stocks and bonds weighted by market value. We also use a TIPS ladder for the risk-free portion, which is our entire allocation to bonds. We show later in this section why the risky portion should be 100% allocated to stocks.

While the changes in market value of a TIPS ladder should be well correlated with those of a conventional bond fund of similar duration,[7] for the purposes of retirement income there are decided advantages to holding a ladder of TIPS instead of bond funds. With the proviso that the individual TIPS are held to maturity, as assumed here, the ladder's cashflows provide predictable, essentially risk-free real income to be matched against some or all of the retiree's spending needs. Bond funds, on the other hand, are volatile. For most runs in our data set bonds have been positively correlated with stocks. The risk-free cashflows from a TIPS ladder provide a stable income floor and are expected to reduce variance of total income more effectively than a comparable allocation to a bond fund. Sharkansky (2024) provides more detail about TIPS, and the advantages of TIPS ladders relative to both TIPS funds and conventional bond funds for this use case.

The TIPS Ladder.

At or before time $y = 0$ a TIPS ladder is constructed to provide a known stream of cashflows from coupon payments and maturing bonds. The only constraint on the sequence of cashflows is the availability of bonds. The ladder could, for example, be set up to provide more income in the years between the onset of retirement and the receipt of Social Security benefits, and less income after Social Security benefits commence. For the purposes of this analysis, we assume that the ladder is constructed to provide constant income every year for 30 years, starting at $y = 1$. At this writing, a constant income 30-year ladder at current yields delivers an annual real income of roughly 4.5% of the initial investment.

Withdrawals from the Risky Assets.

Withdrawals from the risky assets are made each year at time $y \geq 1$. In the general case, the retiree may wish to reserve a portion for a legacy to remain at the end of the time horizon.[8] In the remaining analysis, we assume there is no legacy reserve. The retiree specifies a shape for the sequence of expected future withdrawals by assigning a relative weight to each year's target. For example, if they desire that expected withdrawals are constant for the first 5 years, and in the subsequent 10 years at a constant rate that is double the initial rate, they would specify the weights $w_1 = w_2 = \ldots w_5 = 1, w_6 = \ldots = w_{15} = 2$. Then the withdrawal in year t is to be the following fraction of the risky portfolio value at that time:

$$\frac{w_t}{\sum_{k=t}^{T} \frac{w_k}{(1+d)^{k-t}}} \tag{2}$$

where

- w_k = the weight for year k
- T = final year of plan
- d = annual discount rate

This formula is derived in Appendix C.

If we target constant withdrawals, i.e., every $w_k = 1$, as was implicit in Waring and Siegel (2015), then the withdrawal fraction simplifies to the standard amortization formula:

$$\frac{d(1+d)^{n-1}}{(1+d)^n - 1} \tag{3}$$

where

- d = annual discount rate
- n = number of remaining annual withdrawals, including the current withdrawal. In terms of the preceding equation, $n = T - t + 1$

This is equivalent to the Microsoft Excel expression PMT(d,n,1,0,1).

All of the independent variables may be updated by the retiree at any time per current expected longevity and other needs. The budgeted withdrawal is specifically intended as a budget, not as a rigid directive. In the section below on managing income variability we explore reasons and implications for withdrawing above or below the budget.

The following analysis assumes constant withdrawal targets. In a later section, we explore scenarios for more general time-varying withdrawal targets.

The asset allocation between the TIPS ladder and the risky assets is a fundamental decision. Since income from the TIPS ladder is a risk-free known constant, the risk and reward characteristics of total portfolio income are simple linear functions of the risk and reward of the risky asset income, given the risky fraction of the portfolio. We first address the asset allocation within the risky portfolio and return to the trade-offs in the allocation between TIPS and the risky assets.

Asset Allocation and Management of the Risky Portfolio.

In this section we explore outcomes for the risky portion of the ARVA portfolio, under a range of allocations to stocks and bonds. The bonds considered for the risky portion of the portfolio are assumed to be bond funds, in contrast to the individual TIPS held to maturity in the riskless portion. This is an important distinction. Income is generated from the bond funds by liquidating a portion of the underlying bonds prior to maturity, when market values are volatile. Therefore this income is risky. The bond returns in our data set are a proxy for the total returns of an index fund of U.S. Treasury bonds with a duration of 10 years. For the allocations to stocks vs bonds, we consider 40/60, 60/40, 80/20, and 100/0, with annual rebalancing. We conclude that a 100% equity portfolio historically produced higher withdrawals than the blended portfolios, without increasing downside risk.

For each stock/bond mix the amortization discount rate should be the expected annual real return for the asset mix, for which we use the historical geometric mean real return, as shown in Table 1. Amortizing at a rate above (below) the expected return would overweight earlier (later) withdrawals at the expense of later (earlier) withdrawals.

Figure 2 plots the distribution of yearly withdrawals over a 30-year horizon for 100/0 and 60/40 portfolios amortized at the mix's mean return, assuming a $1 million initial portfolio. Since we have no legacy reserve, the portfolio balance will be drawn down to $0 with the 30th withdrawal. For every run, we compute the average of the 30 annual withdrawals, and the worst case of the 30 withdrawals. Figure 3 shows the Percentile Plots for the average and worst-case withdrawals for each asset mix, amortized with its historical mean return, along with those of a 60/40 portfolio with constant 4% withdrawals.

Appendix D provides additional summary statistics comparing outcomes of the various asset mixes.

Of the asset mixes and amortization rates considered, 100% stock amortized at its historical mean return of 6.9% had the most consistently high median income over time. Withdrawals that fell below the median were fairly similar for 100/0 and the 60/40 and 80/20 blends. While the withdrawals from the 100% stock portfolio would necessarily be more volatile than from the blended stock/bond portfolios, most of the variability is in the upper percentiles. Except for a small number of extreme cases, the poor outcomes (5 percentile and below) for the 100% stock portfolio were comparable to or slightly better than the poor outcomes for the next best blended portfolio. If the more impactful risk is not variability of withdrawals as such, but frequency and severities of low withdrawals, then an all-stock portfolio appears to be less risky in that sense then a blended portfolio. Section 2 of the Online Supplemental Material contains a number of tables and time-series plots which provide insights into the relationships between historical market conditions and the distribution of withdrawals.

Our finding that a 100% stock portfolio would have delivered higher withdrawals historically without greater downside risk than a blended stock/bond risky portfolio is an impactful observation. This challenges the conventional wisdom that an all-equity portfolio is unquestionably too risky and the conventional advice that retirees should hold bond funds. Investors who want to reduce the volatility of their portfolio values and the variability of withdrawals would do well to replace any allocation to conventional bond funds with an equal value in a TIPS ladder held to maturity. The ladder reduces

Table 1. Historical Geometric Mean Annual Real Returns for Mixed Portfolios of Stocks and Bonds (Monthly Rebalancing)

Stock %	0	10	20	30	40	50	60	70	80	90	100
Mean Real Annual Return	2.46%	2.99%	3.51%	4.00%	4.48%	4.93%	5.37%	5.79%	6.18%	6.55%	6.90%

Figure 2. Distribution of Annual Withdrawals Rates for Different Asset Mixes Amortized at Mean Rates of Return, Assuming $1 Million Initial Portfolio

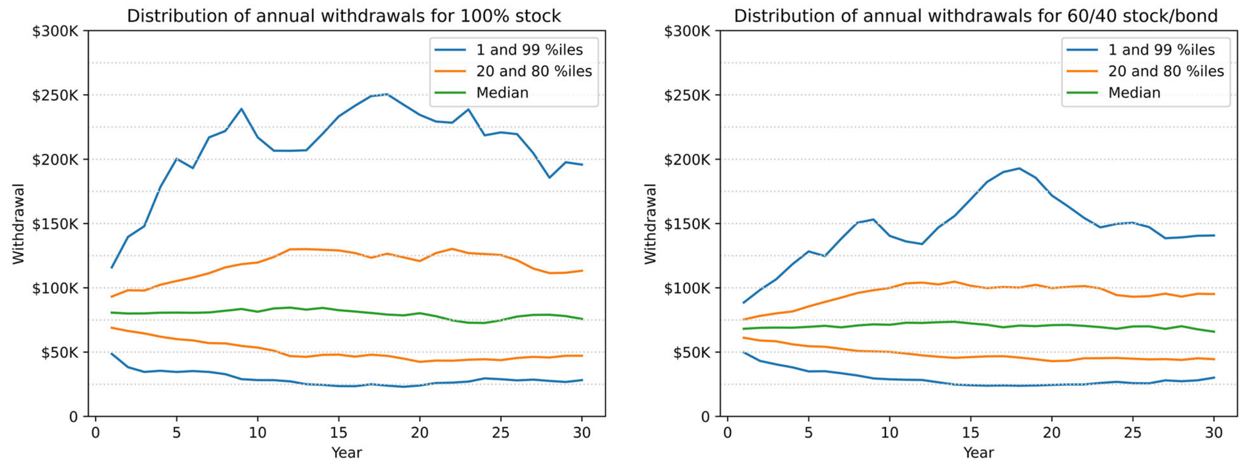
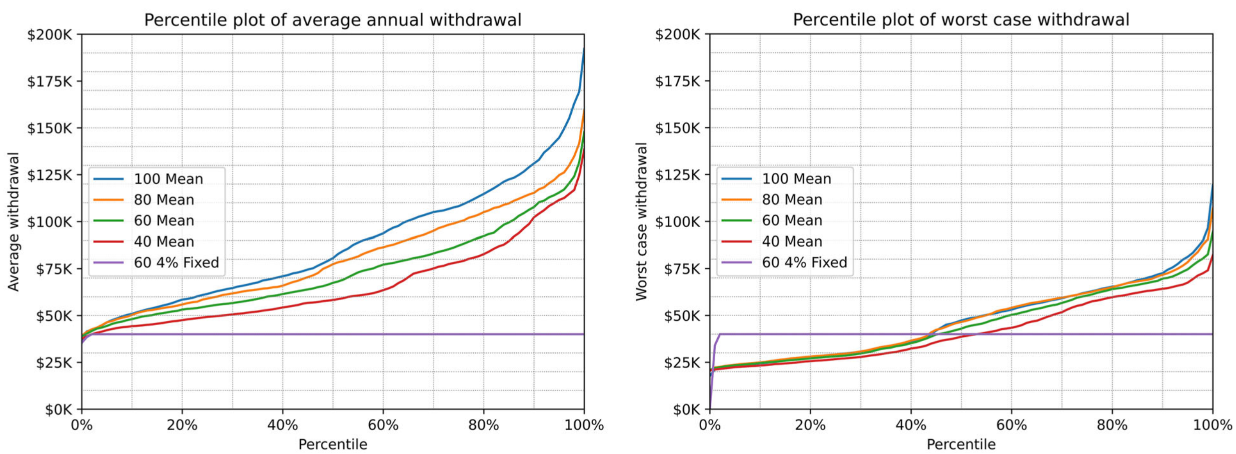


Figure 3. Percentile Plots for Average and Worst-Case Withdrawal Rates, Assuming a $1 Million Initial Portfolio



market value volatility as would a bond fund, but also more effectively stabilizes income.

The empirical distribution of withdrawals contradicts the standard model of stock and bond returns as i.i.d. bivariate (log)normal. While the i.i.d. normal model predicts that the lower quantiles of ARVA distributions from 100/0 should be meaningfully lower than those of 60/40, our empirical finding is that the lower quantiles of distributions from 100/0 are actually slightly higher than those of 60/40. Our exploratory data analysis suggests this is attributable to mean reversion in stock returns over long horizons. Siegel (2023) also reports mean reversion in stock returns. Although short-term stock returns are known to produce more extreme outcomes than the normal distribution, mean reversion implies a narrower range of long-term outcomes than predicted by i.i.d. normality. Section 5 of the Online Supplemental Material provides an example of our findings on mean reversion.

Further analysis of the differences between empirical outcomes vs. those of the classical parametric model is beyond the scope of this article. But since much of contemporary financial planning is based on the classical model, a study of the differences between theory and the historical record, and the implications for financial planning is an important topic for future research.

While the evidence here supports a risky portfolio of 100% stocks amortized at the historical real geometric mean of 6.9%, an investor may allocate and amortize the assets of their ARVA risky portfolio based on their own capital markets forecast while taking advantage of the underlying ARVA concepts which do not make assumptions about the nature of the risky portfolio.

Allocation Between Stocks and TIPS. Given the conclusion that the risky assets should be 100% stocks, we address the question of initial allocation between the stocks portion and the TIPS portion. That decision depends on the retiree's situation. Factors to consider include the retiree's minimum requirement for essential income and target for discretionary income, initial portfolio value, non-portfolio income sources, preferences regarding the trade-offs between the wish for higher withdrawals vs. tolerance for the risk that some withdrawals will be lower than desirable, longevity expectations, and desire for a legacy.

To frame this question as a decision problem in a way that should be meaningful to both advisors and retirees, we consider two metrics, and the historical probability distributions of each:

1. The average annual withdrawal that the investor receives over the retirement period.
2. The worst case of the annual withdrawals that the investor receives during the retirement period.

Figure 4 illustrates these quantities for a 100% stock, 0% TIPS portfolio compared to those for a 50% stock, 50% TIPS portfolio. The annual income assumed for TIPS is that for a 30-year constant ladder of TIPS, which at this writing is $45,141 on a $1 million initial investment, or a 2.08% average real yield.[9]

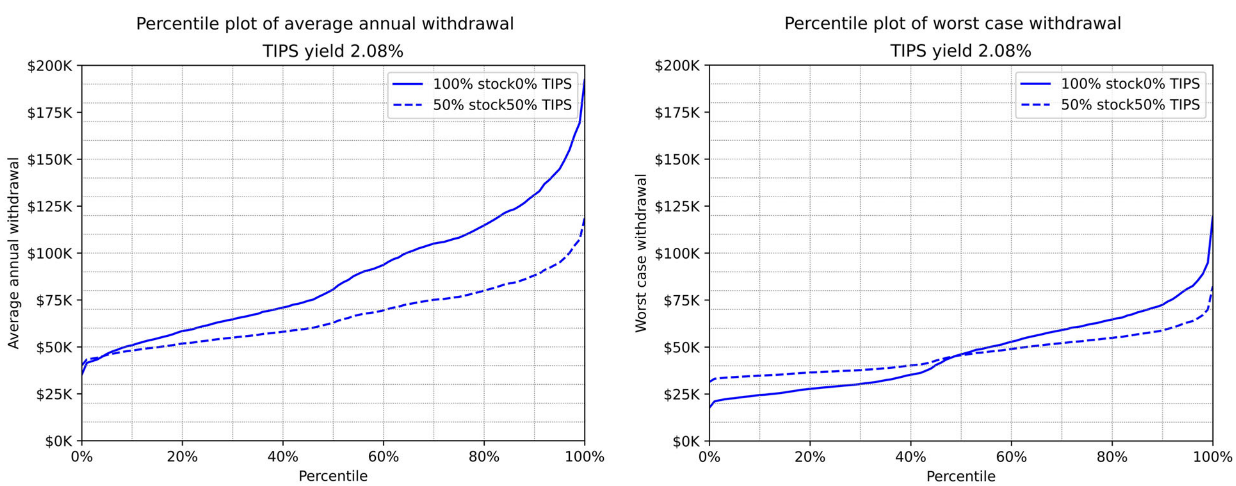
The respective percentiles where the curves cross depend only on the level of TIPS income, and not on the allocated proportions to stocks vs. TIPS.

We express the investor's risk tolerance in terms of the minimum annual withdrawal they are comfortable with and their required confidence level for staying above that threshold. We define the *confidence level* as 100% minus the percentile (e.g., 90% confidence level corresponds to the 10 percentile). For example, an investor might specify that they want every annual withdrawal to be at least $24,000 with 90% confidence. The 10 percentile for the 100% stocks worst case is $24,600, above the lower bound. This investor should accept the risk of a 100% stocks/0% TIPS portfolio and feel 90% confident that they will always be able to withdraw at least at the minimum desirable level. A different investor might desire no less than $34,000 a year with 90% confidence. For this investor, a 50% stocks/50% TIPS portfolio provides the required safety, with the trade-off of nearly certain lower average withdrawals than from 100% stocks.

Generally, for any percentile p and per $1 of initial portfolio value, let the following hold:

$s =$ the fraction of the portfolio allocated to stocks

Figure 4. Percentile Plots for Average and Worst-Case Withdrawal Rates, Assuming a $1 Million Initial Portfolio



T = the annual income from an all-TIPS portfolio

$A_{C,s}$ = the maximum average annual withdrawal from a portfolio with stock fraction s at C% confidence ($1 - C$ percentile); let $A_C \equiv A_{C,1}$

$W_{C,s}$ = the maximum worst annual withdrawal from a portfolio with stock fraction s at C% confidence ($1 - C$ percentile); $W_C \equiv W_{C,1}$.

Then, there is a simple linear relationship between the respective percentiles for a blended stock/TIPS portfolio and the percentiles for an all-stock portfolio:

$$A_{C,s} = sA_C + (1-s)T \text{ and } W_{C,s} = sW_C + (1-s)T \tag{4a,b}$$

If we have a target minimum withdrawal W^*, average withdrawal A^*, and confidence level C in mind, we rearrange the above equations to solve for s and determine the respective allocations to stocks that will provide the given minimum withdrawal, and target average withdrawal at the given confidence level:

$$s = \frac{W^* - T}{W_C - T} \text{ and } s = \frac{A^* - T}{A_C - T} \tag{5a,b}$$

Figure 5 illustrates the linear relationship between stock/TIPS allocation and withdrawals for various confidence levels.

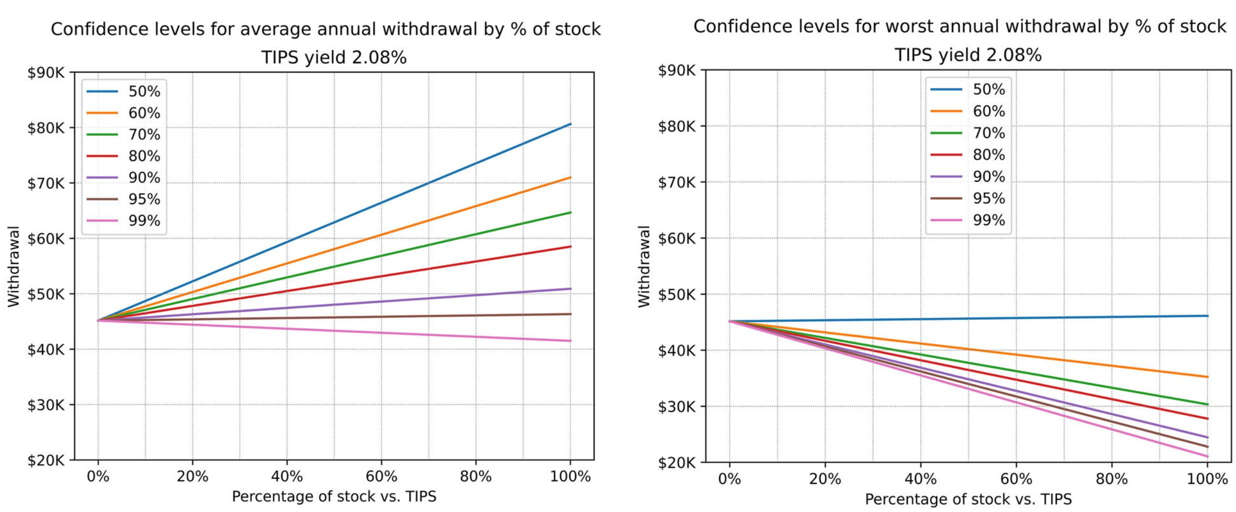
For all reasonable confidence levels (i.e., $\geq$ 50%), the line of worst withdrawal as a function of stock allocation has a negative slope (i.e., the higher the percentage of stock in the portfolio, the worse the worst-case withdrawal). This implies that the above formula for s as a function of W^* provides the *maximum* desirable value of s to ensure that the worst case is not worse than W^*. The line of average withdrawal as a function of stock allocation has a positive slope for all confidence levels < 94% (i.e., the higher the percentage of stock in the portfolio, the higher the average return at every confidence level < 94%). This implies that if an investor is risk averse and requires 94% or more confidence that the average withdrawal be above *any* lower bound, then that investor's portfolio should be 100% TIPS. Another implication is that for confidence levels below 94%, the above formula for s as a function of A^* provides the *minimum* desirable value of s to ensure that the average withdrawal is above A^*.

Sensitivity of Outcomes to the TIPS Yield.

The above analysis is based on current TIPS yields. As noted above, at the time of this writing, the annual real income from a 30-year TIPS ladder with equal annual cashflows is 4.51% of initial value. That imputes an average real yield of 2.08%.[10] How would this strategy perform under different TIPS yields?

The first TIPS were issued in 1997. No 30-year TIPS ladder has yet reached maturity. Simulating TIPS from historical estimated real interest rates would be a plausible alternative, but no ready source exists for such rates prior to 1982. Our analysis is limited to showing outcomes of the stock portfolio paired with pro forma ladders of varying real yields from a reasonable range of real interest rates, and under the assumption that 30-year ex-post stock

Figure 5. Average and Worst Withdrawals for Different Allocations to Stocks and TIPS for Various Confidence Levels. Assuming Current TIPS Yield, $1million Initial Portfolio and Historical Stock Return Distribution



returns are independent of the TIPS yield at time $= 0$.[11]

The range of 10-year real interest rates in the available historical data is between −1.19 and 7.66%. Market yields for TIPS of other maturities have at times been even lower.[12]

In general, we view the considerations regarding the TIPS yield as follows. We focus on the primary use for TIPS, which is to provide floor income and moderate the worst-case outcomes of stocks.

Suppose that a retiree wishes to plan for a minimum acceptable annual withdrawal of W^* with $C\%$ confidence. Let W_C be the maximum annual withdrawal that can be made from an all-stock portfolio at that confidence level (the $100 - C$ percentile of the worst-case withdrawals); i.e., the retiree can be at least $C\%$ confident in the plan only if $W^* \le W_C$. Let Y be the current TIPS yield averaged over the yield curve for a specified ladder, and T_Y be the annual income from the ladder for yield Y, which is computed from the standard annuitization formula. Consider the following cases. We provide numerical examples in Appendix E.

Case 1. $W^* \le W_C$

The desired minimum withdrawal is less than or equal to the worst-case withdrawal for the specified confidence level, i.e., the retiree can be at least $C\%$ confident of being able to withdraw at least W^* every year. Therefore, an all-stock portfolio is adequately safe given the retiree's risk tolerance. Except for the unusual situation that the TIPS yield is so high as to imply a negative equity risk premium, adding TIPS to the portfolio would only reduce the expected upside of the stock holdings. It is unnecessary to add TIPS to the portfolio.

Case 2. $W_C < W^* \le T_Y$

The retiree cannot be $C\%$ confident of being able to withdraw W^* or more from an all-stock portfolio every year. But the TIPS income at this yield is sufficient to not only increase the worst case withdrawal but to increase it enough to be greater than W^*. From Equation 5a, TIPS should be added to the portfolio such that the stock fraction s is as follows:

$$s = \frac{T_Y - W^*}{T_Y - W_C}$$

Case 3. $W_C < T_Y < W^*$

The retiree cannot be $C\%$ confident of being able to withdraw W^* or more from an all-stock portfolio every year. Adding TIPS to the portfolio will improve the worst-case withdrawal, thus reducing risk. But the worst-case will still be less than W^*. The retiree must accept that they cannot be $C\%$ confident that any blend of stocks and TIPS managed by the ARVA strategy will provide a minimum of W^* every year. They must either change their expectations and continue with the plan but recognize that they cannot be as confident of always avoiding a lower than desirable withdrawal; adopt a lower worst-case that they can tolerate with $C\%$ confidence; or switch to a non-ARVA strategy, if one exists that ensures a minimum income greater than W^* with at least $C\%$ confidence, and in consideration of any trade-offs.

Case 4. $T_Y \le W_C < W^*$

As above, the retiree cannot be $C\%$ confident of being able to withdraw W^* or more every year. Since $T_Y < W^*$, the retiree has the same dilemma as in the above case. Furthermore, the income from the ladder at yield Y is lower than W_C, so adding any TIPS to the stock portfolio will only reduce the worst-case income thus increasing the risk, while also reducing expected total income. An all-stock portfolio is more efficient than including any TIPS at this yield.

For every confidence level C there is a "drop-dead" yield Y_C such $T_{Y_C} = W_C$ and for every yield $Y \le Y_C$ TIPS at that yield add no value for any target W^* at that confidence level. The higher the confidence level, the lower the drop-dead yield. The more risk-averse the retiree, the more they need to pay in the form of lower, possibly negative yields, in order to insure against the worst outcomes—down to the drop-dead yield, beneath which the insurance no longer provides protection.

Income Volatility

Many retirees would prefer stable income, at least within predictable bounds, over less predictable and variable income. The PWR math and the budget constraint tell us that there is a cost for even trying to extract stable income from a volatile portfolio. Many financial advisors manage clients' portfolio income using "guardrail" rules to adjust income up or down within formulaic bounds as asset values fluctuate. Perhaps the best known and widely used guardrail strategy is that of Guyton and Klinger (2006) ("GK"). We show that the GK strategy historically would have produced incomes which were not only variable, but were also dominated by ARVA incomes. We conclude this section with approaches for managing the variability of ARVA income.

Comparison with Guyton-Klinger Guardrail Strategy. We analyze the (GK) strategy under the same historical simulation framework. A concise summary of the GK decision rule algorithm is in Section 4 of the Online Supplemental Material. The GK article assessed portfolios whose stock/bond/cash allocations were 65/25/10 and 80/10/10. It concluded that portfolios with stock allocations of at least 65%, and with initial withdrawal rates between 5.2–5.6% were sustainable over 40 years with a 99% probability of success. We compare the GK strategy for 65/35 and 80/20 portfolios (stock/bond only, no cash) against the ARVA strategy for stock/TIPS blends of 65/35 and 80/20, with TIPS yields at both the current yield of 2.08% and the more conservative 1%,[13] which provide annual incomes of $45,141 and $38,748 on a $1 million ladder, respectively. For GK we set the initial withdrawal rate to the maximum reported safe rate of 5.6%, which maximizes the ongoing withdrawals and presents the most favorable comparison for GK.

Figure 6 compares percentile plots for average annual withdrawals and worst-case withdrawals for these configurations. Table 2 summarizes the outcomes of both methods. Additional plots illustrating outcomes are in Section 4 of the Online Supplemental Material.

ARVA compares favorably in historical simulations relative to the GK strategy using the parameters from the GK article. Overall, the ARVA strategy would have provided higher average incomes than GK under the tested configurations. Regarding downside protection, while GK had more favorable median worst-cases than ARVA, ARVA offered more protection from the lower quantiles of worst-cases. GK strategies, on the other hand, banked the foregone retirement consumption into substantial legacies. A retiree with a strong bequest motive and a lower motive for consumption might be attracted to GK and its likelihood of a substantial bequest. An ARVA retiree who wishes to leave a bequest may set aside a specific portion of their wealth and not annuitize it, or may set a long time horizon with the expectation that the residual wealth would be their bequest.

While per-run incomes under GK had lower variance than those under ARVA, they were still variable. Figure 7 shows the ranges of withdrawals under different ARVA and GK parameters. Those with a preference for stable withdrawals would find that increasing the portion of TIPS in an ARVA portfolio would provide more stability and fewer low withdrawals than GK. And GK's stability is attributable to high thresholds for increasing or decreasing withdrawals and even higher thresholds for increasing withdrawals after they have been decreased and

Figure 6. Percentile Plots of Average and Worst-Case Withdrawals for Various Configurations of ARVA and GK

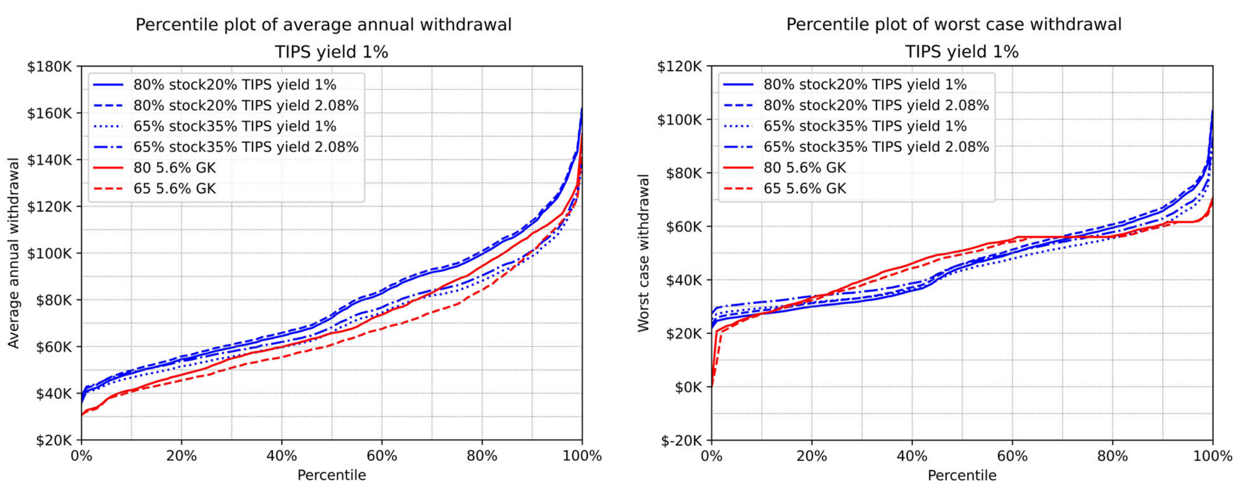


Table 2. Performance of GK Strategy Relative to ARVA, for Comparable Stock Allocations

			ARVA 65/35		ARVA 80/20	
	GK 65/35	GK 80/20	1%	2.08%	1%	2.08%
Median average income	$60,780	$65,693	$65,975	$68,212	$72,258	$73,536
% of runs ARVA average > GK average[a]			62%	71%	80%	84%
Median worst-case income	$49,500	$50,909	$43,541	$45,779	$44,648	$45,926
% of runs ARVA worst > GK worst			48%	57%	50%	57%
Median legacy	$641,922	$818,660	n.a.			
Median ratio legacy/total lifetime withdrawals	33%	38%	n.a.			
Failure rate (premature portfolio depletion)	1.4%	0.1%	n.a.			

[a]Comparing ARVA 65/35 against GK 65/35, and ARVA 80/20 against GK 80/20.

vice versa. Furthermore, there are built-in biases toward lowering rather than raising real incomes, such as the Withdrawal Rule which foregoes cost-of-living increases. Thus the stability does more to protect lower rather than higher incomes.

Managing the Variability of ARVA Income.

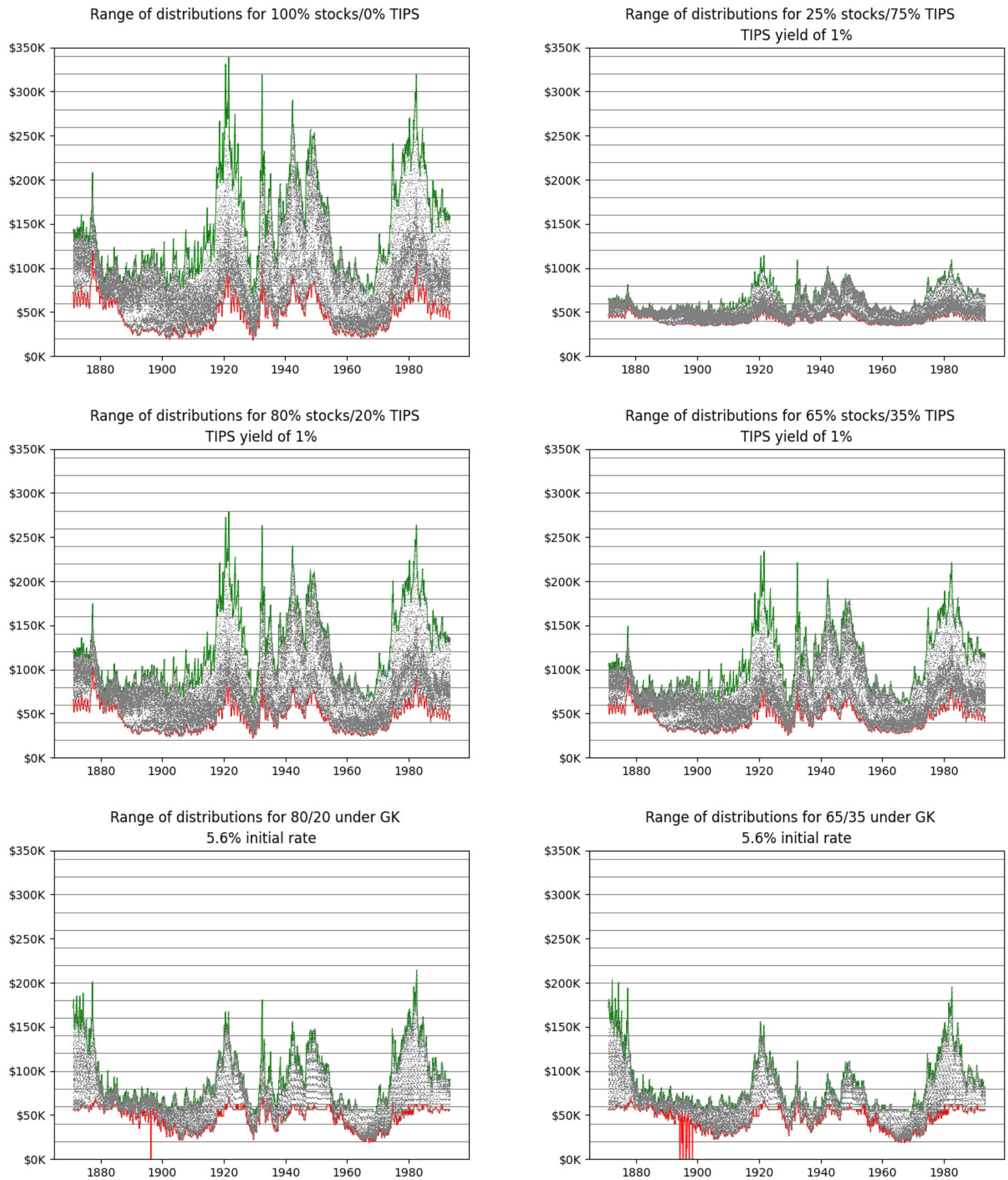
Many retirees and their advisors might like to believe they can smooth their withdrawals from a volatile portfolio. But it's not only GK guardrails that would leave a historical record of income reductions and unintended legacies. As Waring and Siegel (2018) observe succinctly: *Spending rules that don't honor the budget constraint add uncompensated spending risks.* An arbitrary floor risks overspending the budget, which necessarily reduces future income. An arbitrary ceiling risks living beneath one's means. The realistic approach to income variability is not to attempt to eliminate it but to manage it (financially) and adapt to it (behaviorally).

A behavioral approach is to distinguish between priority vs discretionary spending. Set the target for priority income to be relatively low, and set the fraction of TIPS to secure the priority income with a high level of confidence. The variation can be located in the mental account of discretionary income. Instead of setting a high income target where reductions will be painful, designate the discretionary income as a bonus. Frame the plan as having a modest income target, with a smaller likelihood of reductions and the pleasure of frequent bonuses. As Jonathan Guyton points out, retirees have typically spent their working decades learning to adapt to income variation due to life events, setbacks and bonuses (Benz and Ptak 2020).

Financially, the lever that gives the retiree control over income variability is the allocation between stocks and TIPS. The higher the TIPS portion, the less variable the income and the higher its floor. The upper left panel of Figure 7 plots the individual withdrawals from an all-stock portfolio in each run. The green (red) line is the per-run maximum (minimum) withdrawal. The other panels represent the range of withdrawals from portfolios with different proportions of TIPS to stocks—hypothetical with the assumption that TIPS ladders could have been constructed historically. In these plots we assume that the TIPS had an average real yield of 1%. Note that from Eq. 4, the *scale* of the distribution (spreads between maxima and minima) depends only on the fraction of stocks in the portfolio, not on the TIPS yield. For a given stock fraction, the TIPS yield affects only the *location* of the distribution (the minimum withdrawals).

The indicated withdrawals again are not required distributions, but budgeted distributions. If the retiree has a need to spend more than the budget, they may do so, at the obvious cost of reducing the capacity for future withdrawals. A larger than budgeted withdrawal does not reduce the "probability of success." Instead, the ARVA mathematics can be used to illustrate to the retiree the updated distribution of future withdrawals conditioned on a larger present withdrawal and enable him to decide whether to exceed the budget. On the other hand, if the budget exceeds the retiree's current spending needs, they may retain the surplus in the risky assets, or use it to extend or enlarge the TIPS ladder for future riskless income. The ARVA strategy need not be used as a rigid pre-determined formula. It is a framework that enables the retiree to make

Figure 7. Ranges of Historical Withdrawals for Various Allocations between Stocks and Hypothetical TIPS Ladders, Followed by Historical Withdrawals under the Guyton-Klinger Guardrails for Initial 80/20 and 65/35 Portfolios with 5.6% Initial Withdrawal Rate. Green (Red) Points Represent Maximum (Minimum) of the Annual Withdrawals per Period



the most of their spending capacity within describable and acceptable trade-offs that adapt to market realities and the retiree's changing needs and expectations.

Time-Varying Spending Targets

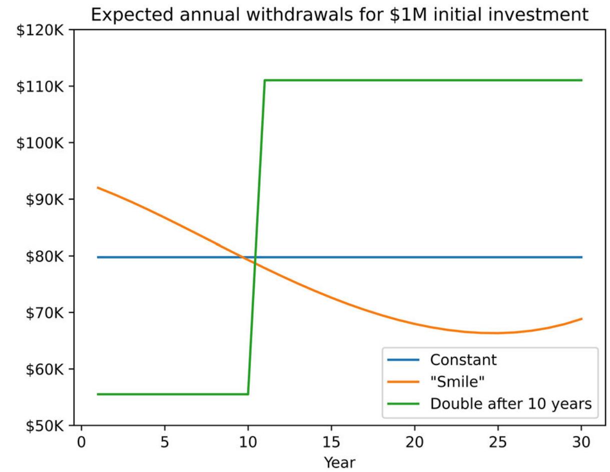
The withdrawal budget formula allows for a non-constant sequence of withdrawal targets. A specific example of non-constant withdrawals would be to plan for an income stream with the shape of Blanchett (2014)'s "retirement spending smile." He found that spending is highest in the early, healthiest, most active years of retirement, then declining over several years as the retiree becomes more sedentary before increasing again to satisfy greater healthcare needs. He found the following formula, by regression of survey data, to predict the annual change in real spending:

$$\Delta = .00008 \times \left(Age^2\right) - (.0125 \times Age) - .0066 \log_e(ExpTar) + 54.6\% \qquad (6)$$

where *ExpTar* is the retiree's after-tax total expenditures.

Figure 8 illustrates the outcomes from applying three different sets of income targets: (1) constant income for 30 years, (2) weights implied by the above "smile" formula for 30 years starting at age 70 with $ExpTar = \$100,000$ (3) constant withdrawals for the first 10 years, followed by 20 years of constant withdrawals at twice the initial level.

Figure 8. Expected Annual Withdrawals for Various Sets of Income Target Weights, Assuming a $1 Million Initial Investment



Managing Longevity Risk

All of our analysis to this point assumed a fixed time horizon of 30 years. That is a reasonably conservative horizon—at the 83 percentile of survival time for a healthy 65-year-old female, the 90 percentile for a healthy 65-year-old male, and 75 percentile for survival time for the longest living of a male/female couple both 65, assuming independence.[14] We explore how a retiree might customize their time horizon, perhaps prioritizing withdrawals for the early years when they are most likely to be alive, or perhaps updating the time horizon dynamically. We discuss longevity risk for the stock portfolio and the TIPS portfolio separately.

Longevity Risk and the Stock Portfolio.

We consider three systematic approaches for managing longevity risk related to the stock portion of the portfolio.

The first approach is to fix a time horizon at the outset, perhaps from a percentile of the retiree's survival time using a demographically suitable mortality table. The percentile would be based on the retiree's risk tolerance and beliefs about their health relative to the mortality table's population. Some retirees might prefer a very long time horizon, say to age 105 (0.2% chance for a 65 year old male to outlive that age) vs. 95 (10% chance of outliving), accepting the trade-off of lower annual income. An insufficiently conservative horizon may lead to either premature portfolio depletion, or force the retiree to extend the time horizon after sufficiently many years so as to cause an undesirably large income reduction.

The second approach, described by Frank, Mitchell and Blanchett (2012a), is to adjust the time horizon dynamically based on percentiles of the survival distribution for the retiree's then current age.[15] For example, they might set the time horizon each year to the 80 percentile. For a male at 65, it would be 27 years, at 75, it would be 18 years, at 80, 14 years. If he were feeling less healthy at 80 he might reduce the percentile from 80% to 50%, in which case the remaining time horizon would be 9 years. If a retiree's risk preferences and perception of their health relative to others in their age cohort remain constant over time, then it follows that their target percentile should also remain constant. But as a person ages, the distribution of their age-at-death moves to the right. Therefore with a constant target percentile the effective planning horizon continually increases beyond what was originally planned for, and expected withdrawals will decline.

A third approach, common to Sharpe (2017) and Kaplan (2020), is for the future income targets to decline with decreasing survival probability but more gradually than survival probability declines. Specifically, in our context, the pre-determined target weights w_k as described in a previous section would be multiplied by the factor p_k^{η} where p_k is the probability of surviving to year k and $0 < \eta < 1$ is a constant, which in Kaplan (2020) is the elasticity of intertemporal substitution. Sharpe (2017) chooses $\eta = \frac{1}{2.9428}$ based on theoretical assumptions about the pricing kernel.

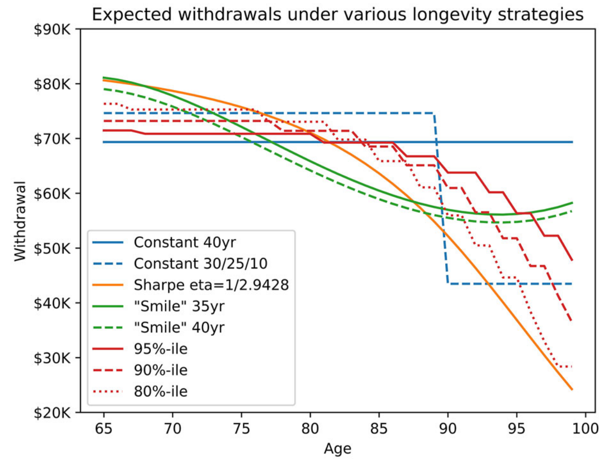
Figure 9 illustrates the trade-offs of the respective approaches for managing longevity risk. Each curve shows the pro-forma withdrawals that each approach would provide a healthy male from age 65 to 100, assuming a $1,000,000 initial portfolio and an ideal asset where the rate of return equals the discount rate. This graph doesn't convey the variance in actual withdrawals, but it is useful for comparing the shapes of expected withdrawals across the approaches. Withdrawals from only the stocks portion are shown for illustration purposes.

The specific approaches are:

- Amortize over 40 years (in which case the portfolio would not be depleted until 105).
- Amortize initially over 30 years. But after 25 years, reset the amortization clock to an additional 10 years instead of 5.
- Amortize over 35 years, with declining weights for declining survival probability per Sharpe (2017).
- Amortize over (35/40) years, with weights calculated per the "smile" of Blanchett (2014). The "smile" pattern doesn't control for longevity as such, and is included for comparison with the declining withdrawals from some longevity approaches. With 40-year amortization the portfolio is not depleted until 105.
- Update the remaining time horizon annually based on the (95%/90%/80%) percentile of the survival distribution. In these cases the portfolio is never depleted.

Approaches which decrease withdrawals with declining survival probability allow for higher withdrawals in earlier years, but the withdrawals would decline rapidly during one's 90s. Extending the amortization period while the plan is underway may produce large drops in withdrawals. Those who envision surviving into their 90s and wish to maintain income during that decade and prepare for greater health care costs might wish to insure themselves in the form of a long initial amortization period. Those who anticipate spending less over time may set target weights in order to front-load early withdrawals along the lines of the Blanchett (2014) "smile." While a reassuringly long amortization period with a high probability of being longer than one's lifetime reduces lifetime spending, the resulting legacy may obviate the need for an explicit legacy reserve. While all of the above shapes for withdrawal sequences influence the expected withdrawals, the variance in actual withdrawals may dwarf the targeted shapes.

Figure 9. Withdrawal Patterns for Various Approaches for Managing Longevity Risk for the Risky Assets, Assuming a $1 Million Initial Investment, and an Ideal Asset Whose Constant Rate of Return Equals the Discount Rate



Longevity Risk and the TIPS Ladder. The longevity considerations for the TIPS ladder are different than those for the stock portfolio. The ladder can be easily constructed to provide cashflows in any desired shape, including to match Blanchett (2014)'s spending pattern or Sharpe (2017)'s adjustments for survival probability. The longevity-related challenge with the TIPS ladder is that the maximum term for existing and anticipated TIPS issues is 30 years. If a retiree desires guaranteed income to extend to, say, 35 years, they could include in each of the first 5 years of the ladder, in addition to funds for that year's spending, sufficient funds to purchase a new 30-Year issue each year. The value at maturity of the delayed purchase is unknown when the ladder is initially constructed, and can only be estimated from one's best forecast of future real rates.

Conclusions

This work contributes to the literature on retirement income by providing extensions and practicable implementation guidance to the ARVA retirement income strategy originated by Waring and Siegel (2015). Our strategy ties the portfolio composed simply of stocks and a TIPS ladder with a formula for spending down the stock portfolio. Our contributions to the ARVA framework include generalizing the model for time-varying income targets, determining the preferred form of the risky asset portfolio, creating guidelines for asset allocation between risky assets and TIPS, and examining the trade-offs of different approaches to managing longevity risk.

We tested variants of the strategy against historical returns, and found that the most promising structure for the risky portion of the ARVA portfolio is 100% U.S. stocks, amortized at 6.9%, the historical geometric mean real annual return on U.S. equities. Although the withdrawals are more variable for the all-stock portfolio than for blended portfolios of stocks and conventional bond funds, the variability was essentially all on the upside. Adding bond funds to the risky portfolio did not mitigate downside risk.

We showed both theoretically and empirically that our method offers a number of advantages relative to retirement income approaches commonly used today, specifically constant rate withdrawal rules (e.g., 4% Rule) and decision rule guardrail approaches. In historical simulations ARVA facilitated a higher level of lifetime spending than such strategies. The upside potential of variable ARVA income is notably higher and its downside risk lower than those of the widely used Guyton and Klinger (2006) guardrail rules, where incomes are also volatile. Another advantage is flexibility, with respect to both market conditions and to the retiree's non-constant consumption goals and changes in their situation. The latter includes flexibility to shorten or lengthen the time horizon with changes in the longevity forecast. Real income from the TIPS ladder is invariant to market conditions, but is easily constructed to provide non-constant incomes. The amortization of the risky asset can be parameterized to produce non-constant expected withdrawals. Prevailing approaches do not readily accommodate varying cashflow goals or changes to the time horizon.

The formula for amortized decumulation of the risky asset ensures that it is depleted precisely at the end of the specified time horizon. For any given time horizon, there is no chance that the ARVA retiree will prematurely run out of money. This is an advantage over constant rate and decision rule methods which are implemented to reduce, but cannot eliminate, a "probability of failure," i.e., premature portfolio depletion. With those approaches, the converse of a low probability of failure is a high probability of always living below one's means while amassing a larger than intended bequest. The scheduled depletion time also has the advantage of giving the ARVA retiree more flexibility with regards to the trade-off between consumption and legacy. They can manage this balance by determining the portion of their assets set aside for their bequest, or by adjusting the length of their time horizon.

The unavoidable trade-off of prioritizing consumption over bequest is that the withdrawals from an ARVA risky portfolio will be variable. Undoubtedly some retires will view the variable income as a disadvantage relative to the promise of a predictable constant income. But such promises cannot be kept—the constant rate and guardrail strategies can also force larger than desirable income reductions. The mathematics of the budget constraint and the Perfect Withdrawal Rate inform us that risky assets necessarily produce risky income. It is a matter of education to help retirees understand the realistic range of available outcomes, and the trade-offs they can choose from.

While the daily market values of a TIPS ladder are volatile, its inflation-adjusted cashflows are predictable and essentially risk-free for the ladder's life, provided that the bonds are held to maturity. The variance of the income of a blended stock/TIPS portfolio, is simply the variance of income from an all-stock ARVA times the fraction of stock in the portfolio. One can conveniently dampen the volatility of ARVA income simply by increasing the fraction of the portfolio allocated to the TIPS. TIPS are a more reliable vehicle for reducing income volatility than a conventional bond fund. A bond fund's income would always be volatile, and in most time periods would have been positively correlated with income from stocks.

Even if a retiree chooses to decumulate their risky assets using an approach other than ARVA, incorporating a TIPS ladder into their retirement plan merits serious consideration—especially at this writing when real yields are above their historical average, and a 30-year constant ladder delivers a higher rate of real income than the 4% Rule.

The implementation and management of an ARVA strategy are straightforward. The TIPS ladder needs to be constructed at the beginning of the retirement period. There are currently a number of free online calculators which use recent market prices to compute the exact TIPS issues and quantities to purchase in order to produce the user's desired sequence of real incomes.[16] Once the ladder is constructed, the risk-free cashflows from principal repayments and coupons will be automatically deposited into the retiree's brokerage account for the life of the ladder without any further effort by the retiree. In order to amortize the risky asset, the retiree need only specify the desired "shape" (relative proportions) of the expected income targets. The periodic withdrawals as a percentage of market value at withdrawal are easily calculated in a spreadsheet using Eq. (2). No manual rebalancing between TIPS and the risky asset is needed. If the risky asset is 100% stock, then rebalancing is a non-issue.

Since an ARVA does not run out of money, the notion of probability of success (failure) as often used today to formulate and explain a retirement plan is not applicable. The relevant notion of risk pertains to the distribution of withdrawals and probabilities of maintaining income of a certain level. A common approach today is for an advisor to explain a retirement plan along the lines of providing $100,000 every year with a 90% probability of success. An advisor might instead explain an ARVA plan in terms of a reliable base income of $80,000 a year, along with a variable bonus that has 90% confidence of delivering both an average of $40,000 and a minimum of $25,000 every year, and no historical precedent for ever providing less than $15,000 in any year.

Editor's Note

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Notes

1. Recipient of the *Financial Analysts Journal's* 2015 Graham and Dodd Top Award.
2. We use the word "retiree" to mean either an individual or a couple.
3. We omit non-U.S. equities from our analysis because there is no available data set which provides non-U.S. equity returns with the same frequency and time period as our U.S. equity data. Historical evidence on the question whether global equity portfolios have had higher risk-adjusted returns than U.S.-only portfolios is mixed and time-dependent. Dimson, Marsh, and Staunton (2021), using a sparser data set than ours, found that between 1900–2020 U.S. equity returns were substantially higher and less volatile than non-U.S. returns. McQuarrie (2024) reported similar findings from a longer data set. Correlation of monthly returns between the S&P 500 and MSCI EAFE from 1991 to present has been 0.8.
4. Basic analysis of our data set shows that univariate returns are neither normal nor log-normal; long-term stock returns evidence mean-reversion; bond returns are serially correlated; the correlations of monthly stock and bond returns over the rolling 360-month periods have ranged between −0.19 and +0.37.
5. The PWR is "perfect" in the sense that it is the maximum *constant* withdrawal rate. There can be non-constant withdrawal sequences that deplete the portfolio upon the last withdrawal and where the sum of withdrawals over the entire period exceeds the sum under the PWR.
6. We discuss in a later section how to construct a TIPS ladder so that it can eventually be extended beyond the original 30 years. But conditioned only on information available at inception, such income more than 30 years after inception is not riskless.
7. The correlation of the monthly Bloomberg Treasury and Bloomberg TIPS total return indexes from the 1997 introduction of TIPS to the present has been 69%.
8. The actual legacy will be stochastic, so any target legacy is aspirational. The reserve could be the estimated present value of the target for a projected time horizon.

9. Yield as of February 24, 2025. There are not, at the time of this writing, any TIPS issues that mature during the years 2036–2039. In order to provide the full income target during those years, our pro-forma ladder includes Treasury Zero Coupon Bonds that mature in each of the respective years. To mitigate inflation risk, their nominal principal values are set to be the target real value inflated by 3% annually. The Treasury's current practice is to issue a 10-Year TIPS every January and July, and a 30-Year every February, so it is anticipated that by 2029 there will be TIPS issues maturing every year for 30 years out.

10. Current market yields on the individual issues across the yield curve range from 0.71% to 2.32%.

11. Our data shows that period yields on 10-Year nominal Treasuries at $y = 0$ have been correlated with the periods' subsequent worst-case withdrawals ($\rho = 0.32$) and average withdrawals ($\rho = 0.20$). The association is acceptably small for this illustration.

12. The possibility of a negative real yield raises the question of if and when it would make sense for a retiree to invest in an asset that is guaranteed to steadily lose purchasing power. It could in fact make sense for a risk averse investor to accept a predictable and tolerably small loss as insurance against the risk of an even larger loss.

13. 1% is at the 27 percentile for real yields since 1982. The corresponding nominal yield in that period was 3%, at the 22 percentile in the entire data set.

14. Survival probabilities in this section are derived from the Society of Actuaries RP-2014 Mortality Tables. https://www.soa.org/resources/experience-studies/2014/research-2014-rp/.

15. Frank, Mitchell, and Blanchett (2012a) appear to use the term *percentile* to mean survival probability, e.g., what they refer to as "10 percentile" is the age beyond which the survival probability is 10%. We refer to that same age as the 90th percentile, since 90% of the relevant population would have died before that age.

16. A calculator developed by the author for free public use is at https://personalfund.com/tips-ladder/.

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Appendix A. Literature Review Summary Table

Table A1 lists the above papers in chronological order of publication with "x" indicating which of the features characterize the strategy(ies) the article considers. Note the evolution over time from decision rules based on probability of failure towards more dynamic and flexible strategies.

Table A1. Characteristics of Retirement Income Strategies in the Literature

Article	Focuses on minimizing probability of failure (portfolio depletion)	Employs decision rules or "guardrails"	Designed to exhaust portfolio precisely at end of pre-determined time horizon without unintended legacy	Express configuration of non-constant consumption targets	Mortality updating / withdrawals decline with survival probability	Employs dynamic programming techniques	Dynamic recalculation of asset allocation
Bengen 1994	x						
Bengen 2001	x	x					
Guyton and Klinger 2006	x	x					
Frank, Mitchell, Blanchett 2011	x	x					
Frank, Mitchell, Blanchett 2012a	x	x			x		
Frank, Mitchell, Blanchett 2012b	x	x		x	x		
Blanchett, Kowara, Chen 2012	x	x	x		x		
Zolt 2013	x	x					
Waring and Siegel 2015			x				
Sharpe 2017			x		x		
Kaplan 2020			x	x	x		x
Kobor and Muralidhar 2020						x	x
Blanchett 2023				x		x	
De Santis 2023				x		x	x
This article			x	x			

Appendix B. Data and Simulation Methodology

The Shiller data set includes U.S. stock and bond returns and a price index. U.S. stock returns are represented by the total returns, with monthly reinvested dividends, of the S&P Composite/500 1926–2023, and from Cowles and Associates 1871–1926. Bond interest rates and total returns are for 10-Year U.S. Treasuries. The price index is the Consumer Price Index for All Urban Consumers (CPI-U) from the U.S. Bureau of Labor Statistics) 1913–2023 smoothly joined to Warren and Pearson's price index 1871–1913. The monthly nominal stock and bond returns are converted into real returns by the formula:

$$r_r = (1 + r_n)/(p_e/p_s) - 1$$

where r_r, r_n are the real and nominal returns, and p_s, p_e are the starting and ending price levels. All returns reported herein are inflation-adjusted. All reported dollar amounts are inflation-adjusted relative to the applicable starting date.

Summary statistics for the inflation-adjusted stock and bond returns in the data set follow. Plots of the respective time series are provided in Section 1 of the Online Supplemental Material.

Readily available data on historical real interest rates is limited. Our source is the Federal Reserve Bank of St. Louis https://fred.stlouisfed.org/, which provides daily market yields on each TIPS issue since their inception in 1997. It also provides estimated 10-Year real interest rates monthly from January 1982 to the present and daily 10-Year common maturity market yields, both nominal and inflation-indexed, starting January 1962 and January 2003, respectively.

Simulation Algorithm

Similar to a Monte Carlo simulation, we draw N sequences of 30 years of joint real returns on stocks and bonds. But instead of generating the returns randomly, we draw these from the historical record. Starting every month from January 1871 through

Table B1. Summary Statistics for Rolling 30-Year and 3-Year Periods of Stock and Bond Annual Returns

Geometric Mean Annual Returns					
Stocks					
1871–2023			6.90%		
	Min	5%-ile	50%-ile	95%-ile	Max
30 year periods	1.9% Jun 1902	4.0%	6.7%	9.1%	11.2% Jun 1932
3 year periods	−35.2% Jul 1929	−9.7%	7.6%	25.2%	39.2% Sep 1926
Bonds					
1871–2023			2.46%		
	Min	5%-ile	50%-ile	95%-ile	Max
30 year periods	−1.5% May 1940	−0.01%	2.1%	5.6%	6.9% Sep 1981
3 year periods	−13.2% Jan 1917	−6.9%	2.5%	13.1%	19.2% Aug 1983

continued

Table B1. Summary Statistics for Rolling 30-Year and 3-Year Periods of Stock and Bond Annual Returns (continued)

Standard Deviation of Monthly Returns					
Stocks					
1871–2023			4.09%		
	Min	5%-ile	50%-ile	95%-ile	Max
30 year periods	3.1% Jan 1871	3.3%	3.6%	6.1%	6.2% Nov 1916
3 year periods	1.5% Jan 1992	2.3%	3.3%	6.4%	13.6% Sep 1930
Bonds					
1871–2023			1.67%		
	Min	5%-ile	50%-ile	95%-ile	Max
30 year periods	0.8% Jan 1923	0.9%	1.6%	2.3%	2.4% Jul 1979
3 year periods	0.3% Oct 1962	0.5%	1.4%	2.7%	4.4% Nov 1979
Correlation of Monthly Returns of Stocks and Bonds					
1871–2023			13.3%		
	Min	5%-ile	50%-ile	95%-ile	Max
30 year periods	−19% Aug 1991	−8%	18%	33%	37% May 1898
3 year periods	−77% Mar 2010	−37%	18%	57%	69% Jan 1993

For the Min and Max, the starting dates of the respective periods are shown.

June 1993, we take the returns from the ensuing 30 12-month time spans in chronological order. Thus, we have N = 1,470 overlapping time periods.

To simulate the withdrawal sequence for each period, we let $y = 0$ be the beginning of the first year of the period. At that time the retiree decides to retire one year hence, and the simulated portfolio is constructed. In most of our illustrations the starting value is $1,000,000. The time $y = 1, \ldots, 30$ is considered the end of year y. At time, $y \geq 1$ the y^{th} annual return is applied to the net portfolio from time $y - 1$, and then the y^{th} withdrawal is made (for spending in year $y + 1$) from the resulting portfolio value per the ARVA formula. Unlike in some spending rule studies, where the initial withdrawal is deterministic and identical in every simulation run, in this work the first withdrawal is made after the first year of returns, and therefore stochastic. We define the Legacy to be the value of the portfolio at $y = 30$ net of the 30^{th} withdrawal.

Appendix C. Derivation of the Withdrawal Formula for the Risky Asset

We start with the budget constraint as an explicit equation. Let P_u^t be the present value at time t for the withdrawal at time u. Let F_t be the future value of the time t withdrawal, i.e., $F_t = P_t^t$. Let P^t be the total value at time t before any withdrawals are made. Then at each t, $P^t = P_t^t + P_{t+1}^t \ldots + P_T^t$, where T is the final time.

At the outset we have $P^0 = P_1^0 + P_2^0 \ldots + P_T^0$. We assume that $E[F_t] = P_t^0(1+d)^t$, for expected return d, but in fact for every $t \le u \le T$, $P_u^t = a_t P_u^0$, where a_t is the realized appreciation between times 0 and t. We choose a *shape* of desired withdrawals such that every expected withdrawal is expressed as a multiple of the expected first withdrawal, i.e., $E[F_t] = w_t \cdot E[F_1]$ for some constant w_t (e.g., if we want the expected withdrawal at t to be double the expected first withdrawal, we set $w_t = 2$). Then we set the initial present values to correspond to their expected future values.

Since $E[F_t] = P_t^0(1+d)^t = w_t \cdot E[F_1] = w_t P_1^0(1+d)$, then

$$P_t^0 = \frac{w_t P_1^0}{(1+d)^{t-1}}$$

Thus at t, the fraction f_t to be withdrawn from the portfolio is

$$f_t = \frac{F_t}{P^t} = \frac{P_t^t}{\sum_{k=t}^T P_k^t} = \frac{a_t P_t^0}{\sum_{k=t}^T a_t P_k^0} = \frac{\frac{w_t P_1^0}{(1+d)^{t-1}}}{\sum_{k=t}^T \frac{w_k P_1^0}{(1+d)^{k-1}}} = \frac{w_t}{\sum_{k=t}^T \frac{w_k}{(1+d)^{k-t}}}$$

Appendix D. Comparison of Different Stock/Bond Mixes for the Risky Asset

Rows explained as follows:

- *Median average withdrawal: The median of the average withdrawals, as defined above.*
- *Median standard deviation of withdrawals: For each run the standard deviation of the 30 withdrawals is calculated. This is the median over all those values.*
- *Superiority of average for 100% stock: The percentage of runs for which the average withdrawal from the 100% stock portfolio was greater than the average withdrawal from the indicated blended portfolio.*
- *Always $\geq$ \$40,000: The percentage of runs for which all 30 withdrawals were above \$40,000.*
- *Mean # of years $\leq$ \$40,000: The mean, over all runs, of the number of years in the run when the withdrawal was below \$40,000.*
- *Always $\geq$ \$60,000: The percentage of runs for which all 30 withdrawals were above \$60,000.*
- *Mean # of years $\geq$ \$60,000: The mean, over all runs, of the number of years in the run when the withdrawal was above \$60,000.*
- *Median minimum withdrawal: The median of the worst case withdrawals, as defined above.*
- *Superiority of worst for 100% stock—the percentage of runs for which the worst case for the 100% stock portfolio was higher (not as bad) as the worst case for the blended portfolio.*
- *Mean advantage of 100% at worst: The mean, over all runs, of the worst case for the 100% stock portfolio less the worst case of each blended portfolio.*
- *Worst worst case: The lowest of all yearly withdrawals over all runs.*

Table D1. Budget Withdrawal Rates (as Percentage of Each Year's Current Portfolio Value) When Portfolio Withdrawals Are Amortized for Remaining Years at the Respective Historical Geometric Mean Return

	Asset Mix		
Years	60% stock	80% stock	100% stock
30	6.44%	6.97%	7.46%
25	6.99%	7.49%	7.96%
20	7.86%	8.33%	8.76%
15	9.37%	9.81%	10.21%
10	12.51%	12.91%	13.26%
9	13.57%	13.95%	14.30%
8	14.90%	15.27%	15.61%
7	16.62%	16.98%	17.30%
6	18.92%	19.26%	19.57%
5	22.15%	22.47%	22.75%

Table D2. Comparison of Withdrawals for Various Asset Mixes Amortized at Historical Mean Rates of Return

	All Stock	80/20	60/40	40/60
Median average withdrawal	$80,636	$77,422	$67,453	$58,267
Median standard deviation of withdrawals	$24,155	$19,198	$14,719	$12,183
Superiority of average for 100% stock		82.2%	89.3%	93.3%
Always $>$ $40,000	55.1%	54.8%	53.0%	44.2%
Mean # of years $<$ $40,000	3.3	3.2	3.4	4.1
Always $>$ $60,000	28.5%	28.3%	25.6%	19.0%
Mean # of years $>$ $60,000	21.8	21.4	19.9	15.8
Median minimum withdrawal	$46,123	$45,317	$41,994	$37,361
Superiority of worst for 100% stock		52.1%	61.2%	71.3%
Mean advantage of 100% at worst		$131	$2,230	$5,968
Worst worst case	$17,768	$20,716	$20,828	$20,681

Appendix E. Examples of Sensitivity to TIPS Yield

Numerical examples of TIPS yield sensitivity for an initial portfolio value of $1,000,000 and $C = 80\%$. Dollar values assume the historical distribution of stock returns, under which $W_C = \$27,756$.

Case 1. If the retiree's $W^* = \$25,000$ then an all-stock portfolio is safe within their risk tolerance parameters, and no TIPS are needed.

Case 2. If the retiree's $W^* = \$30,000$. A TIPS yield of -0.67% would produce $30,000 annual income from a $1,000,000 ladder. At that yield, the retiree could hold an all-TIPS portfolio and not worry about falling below the acceptable worst case. At any higher yield the retiree could hold both TIPS and stocks to avoid unacceptably low incomes with C% confidence.

A TIPS yield of $+0.67\%$ would produce $36,900 annual income on a $1,000,000 ladder. Annual income from a roughly 75% stock/25% TIPS portfolio at this yield would never dip below $30,000 with the specified confidence level.

Case 3. Example: The retiree's $W^* = \$38,000$ and the TIPS yield is 0.5%, which produces $36,000 in annual income on a $1,000,000 ladder. The retiree's goal to extract the higher expected total wealth from the ARVA while never experiencing a year with a withdrawal less than $38,000 cannot be achieved with 80% confidence when TIPS are at the given yield. However, a, say, 25% stock/75% TIPS portfolio would facilitate a $34,000 annual income with 80% confidence and the opportunity to receive higher expected total income. A yield of 0.87% or higher, however, would provide enough income to meet or exceed $38,000.

Case 4. Example: The drop-dead yield for $C = 80\%$ is −1.11%. For $C = 90\%$, $W_C = \$24,459$, and the drop-dead yield is −1.89%.